# New tools for the construction, analysis and interpretation of social indicators based on ordinal variables

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# Premise

Issues in creating indicators representing phenomena, for evaluation and governance aims.

# Premise → 1
## between accuracy and ambiguities

Socio-economic phenomena
can be measured and represented
by means of

- “hard” approaches (e.g., financial analysis) → *sometimes*
- “soft” approaches → *often*

# PREMISE → 1
## BETWEEN ACCURACY AND AMBIGUITIES

This because the true nature of
**socio-economic phenomena**

➜

very often **qualitative** and **ordinal**

⇩

Ordinal data are
the true expression of real phenomena

not just a rough approximation of true precise,
yet non-observable, variables;

# Premise → 1
## between accuracy and ambiguities

Ambiguities and nuances of
socio-economic phenomena
are not an obstacle to be removed;
they often are what really matters.

# Premise → 2
## between objectivity and subjectivity

Defining and using data in socio-economic statistics inevitably involves **subjectivity**.

This is also true for decision making purposes.

# Premise → 2
## between objectivity and subjectivity

This is not an issue in itself, since the knowledge process always involves

- "objectivity", in observational methods
- "subjectivity", in definitions and other choices (conceptual framework, data definitions, analytical approaches, …)

The epistemological research of the last century clearly showed as objectivism cannot account for the knowledge process (just like idealism)

# Premise → 2
## between objectivity and subjectivity

So, using *subjectivity* is completely consistent with the aims of the socio-economic analysis.

The real issue is not whether using subjectivity or not; it is how to consistently combine subjectivity and the need to observe and analyse data consistently and objectively.

# PREMISE → 2
## BETWEEN OBJECTIVITY AND SUBJECTIVITY

Subjective choices are unavoidable and their use is fully justifiable.

Real issue → *how to build a sound methodological process, where the subjective choices are clearly stated, while their consequences are worked out in a formal and unambiguous way*.

↓

Final results will be clearly interpretable and the role of subjective inputs and sound formal computations can be clearly distinguished and understood.

# Premise → 3
# data metrics

When dealing with ordinal data, common statistical practice is not quite clearly.

With the aim of pursuing metric analysis out of non-metric data, a lot of arbitrary choices are often taken in data analysis

🡻

Final result depends upon subjective choices.

# Premise → 3
## data metrics

In the end, it is not clear whether the results reflect real facts and sound interpretations or are induced by arbitrary methodological choices (e.g., how non-metric data are turned into metric scales).

## *INDICATORS CONSTRUCTION*

1. → **the process**
2. → **methodological critical issues**
3. → **the alternative approach**

## *AN APPLICATION*

4. → **traditional approch**
5. → **alternative approach**
6. → **state of the art and future perspectives**

# The process

Two phases:

I. CONCEPTUAL DEFINITION (FRAMEWORK AND STRUCTURE)

II. ANALYTICAL TOOLS AND STRATEGIES

# The process

## I. CONCEPTUAL DEFINITION (FRAMEWORK AND STRUCTURE)

| Scope | Questions | | Answers |
|---|---|---|---|
| | *How can be…* | | |
| **DEVELOPING INDICATORS** | ⇨ *… the picture conceptually designed?* | ➔ ① | **through a hierarchical design** ↓ |
| | ⇨ *… the indicators conceptually defined?* | ➔ ② | **by a model of measurement** ↓ |
| | ⇨ *… the picture conceptually organized?* | ➔ ③ | **by developing a system of indicators** |

# The process

## hierarchical design ← ❶

Indicators should be developed through a logical modelling process conducting from concept to measurement. Given its features, this logical design is defined *hierarchical*, since each component is defined and finds its meaning in the ambit of the preceding one.

Conceptually, the hierarchical design is characterized by the following components:

(i) the conceptual model,

(ii) the areas to be investigated,

(iii) the latent variables, and

(iv) the elementary (basic) indicators.

# The process

## model of measurement ← ❷

A further component of the hierarchical design definition is represented by the relationships between:

- *Latent variables and the corresponding indicators*: these relations define the **model of measurement**. Consistently with the measurement model, also the relationship between the *elementary indicators* should be defined.
- *Latent variables* for a given area: these relations are defined in the ambit of the conceptual model and identify the structural pattern (**modelling indicators**).

# The process

## system of indicators ← ❸

A **system of indicators** represents the fulfilment of the conceptual framework. Moreover, it

- offers an effective organizational context, relying on methodological supports and allowing data to be managed;
- allows structured and systematic data to be used, observed in long-term longitudinal perspective. This is particularly demanding with reference to subjective data, which require a great use of resources (beyond a solid survey research methodology).

# The process

## II. ANALYTICAL TOOLS AND STRATEGIES

| Scope | | Questions | | | Answers |
|---|---|---|---|---|---|
| | | *How can be…* | | | |
| **MANAGING THE COMPLEXITY** | ⇨ | *… the observed picture simplified?* | ➔ | ④ | **by reducing the complexity of data structure** |
| | | | | | ➔ |
| | ⇨ | *… the whole picture captured?* | ➔ | ⑤ | **by combining indicators** |
| | | | | | ➔ |
| | ⇨ | *… the whole picture explained?* | ➔ | ⑥ | **by modelling indicators** |

# The process

## reducing the complexity of data ← ❹

The consistent application of the hierarchical design produces a complex data structure (elementary indicators, cases, variables, areas, etc.). In order to manage the complexity:

- aggregating elementary indicators for each variable → re-constructing the conceptual variables consistently with the approach (reflective or formative) adopted at micro level (*construction of synthetic indicators*)
- aggregating units/cases: leading information observed at micro-level to the proper macro level (*definition of macro-units*).

# The process

## reducing the complexity of data ← ❹

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | columns | → | indicators | | each case | | a synthetic value |
| the aggregation goes through | | | | with reference to | | in order to obtain | |
| | rows | → | units | | each indicator | | a macro-unit |

# The process

## combining indicators ← ❺

In some occasion, the complexity of the system of indicators may require the indicators allowing for more comprehensive measurement, in order to (Noll, 2009)

- answer the call by 'policy makers' for condensed information
- improve the chance to get into the media (compared to complex indicator systems)
- make multi-dimensional phenomena uni-dimensional
- compare situations across time more easily
- compare cases (e.g. nations) in a transitive way (ranking)
- to observe and record change across time, difference between groups of population or comparison between cities, countries, ...

Dashboards or composite indicators → useful approaches for summarising indicators.

# The process

modelling indicators ← ❻

This stage is aimed at analysing different aspects of the defined model (e.g. objective and subjective indicators) in order to find explanation by identifying the proper analytical approaches.

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**3. → the alternative approach**

## *AN APPLICATION*

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**6. → state of the art and future perspectives**

# Methodological critical issues

social indicators construction

↓

consolidated tradition

however

critical issues remained unsolved and unsettled

# Methodological critical issues

with reference to difficulty in dealing with data which

- refer to a complex reality
- are ambiguous and softened
- are multidimensional
- are dynamic and evolutionary
- are qualitative also when quantitatively measured
- contain errors and approximations
- are sensitive

# Methodological critical issues

new challenges and perspectives

to improve technical tools strategies

with reference to

- reducing data structure in order to aggregate
  - units
  - indicators
- combining indicators
- communicating the “picture” obtained through the indicators (correctly and significantly representing and showing results).

# Methodological critical issues

new challenges and perspectives

to improve analytical tools and strategies

which should take into account

- nature of data → generally ordinal
- process and trends of phenomena → monotonic

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# Introduction

The particular application illustrated here is aimed at

comparing

the traditional and the alternative approach

to reduce the complexity of data structure,

by using subjective and objective data provided by the European Social Survey project.

In particular, we selected the following information:

# Introduction

| European Social Survey | | | | | | |
|---|---|---|---|---|---|---|
| Area | Variable | Items | Item number<br>R1 (2002) | | Scaling technique | Model of measurement |
| Politics | Self-placement | placement on left-right scale | B28 | LRSCALE | 0 (left) – 10 (right) | |
| Subjective aspects | Life satisfaction | how satisfied with life as a whole | B29 | STFLIFE | 0 (extremely dissatisfied) – 10 (extremely satisfied) | |
| Immigration and asylum issues | Acceptance of immigration: allow | many/few immigrants of same race/ethnic group as majority | D4 | IMSMETN | 1. allow many<br>2. allow some<br>3. allow a few<br>4. allow none<br>to come and live here | reflective |
| | | many/few immigrants of different race/ethnic group from majority | D5 | IMDFETN | | |
| | | many/few immigrants from richer countries in Europe | D6 | EIMRCNT | | |
| | | many/few immigrants from poorer countries in Europe | D7 | EIMPCNT | | |
| | | many/few immigrants from richer countries outside Europe | D8 | IMRCNTR | | |
| | | many/few immigrants from poorer countries outside Europe | D9 | IMPCNTR | | |
| Socio-demographic profile | Income | feeling about household's income nowadays | F31 | HINCFEL | 1. living comfortably<br>2. coping<br>3. difficult<br>4. very difficult<br>on present income | |

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# Traditional approach

## First stage: synthesizing indicators at individual level

**Goal**: synthesizing indicators related to each variable consistently with the adopted model of measurement (reflective or formative).

# Traditional approach

## First stage: synthesizing indicators at individual level

First level

Acceptance of immigration

| Variable | Items | Item code | Loading |
|---|---|---|---|
| Acceptance of immigration: allow | many/few immigrants of same race/ethnic group as majority | IMSMETN | .8 |
| | many/few immigrants of different race/ethnic group from majority | IMDFETN | .9 |
| | many/few immigrants from richer countries in Europe | EIMRCNT | .7 |
| | many/few immigrants from poorer countries in Europe | EIMPCNT | .9 |
| | many/few immigrants from richer countries outside Europe | IMRCNTR | .8 |
| | many/few immigrants from poorer countries outside Europe | IMPCNTR | .9 |
| | | **Total variance explained (%)** | 70 |
| | | **Cronbach's alpha** | .94 |

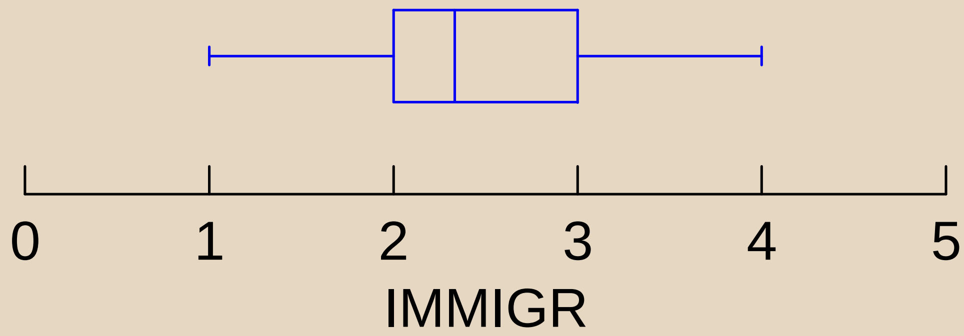
# Traditional approach

## First stage: synthesizing indicators at individual level

First level

Acceptance of immigration

| Synthetic score (IMMIGR) | |
|---|---|
| Minimum | 1.0 |
| Maximum | 4.0 |
| Median | 2.3 |
| Mean | 2.4 |
| Standard Dev | 0.7 |
| Skewness | 0.1 |
| Kurtosis | -0.2 |




1 (allow many) – 4 (allow none)

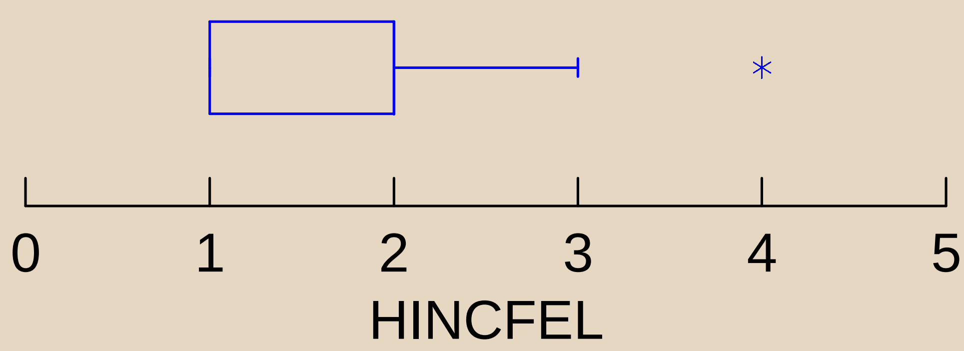
# Traditional approach

## First stage: synthesizing indicators at individual level

First level

feeling about household's income nowadays

| Synthetic score (HINCFEL) | |
|---|---|
| Minimum | 1.0 |
| Maximum | 4.0 |
| Median | 2.0 |
| Mean | 2.0 |
| Standard Dev | 0.8 |
| Skewness | 0.6 |
| Kurtosis | -0.2 |




1 (comfortably) – 4 (difficult)

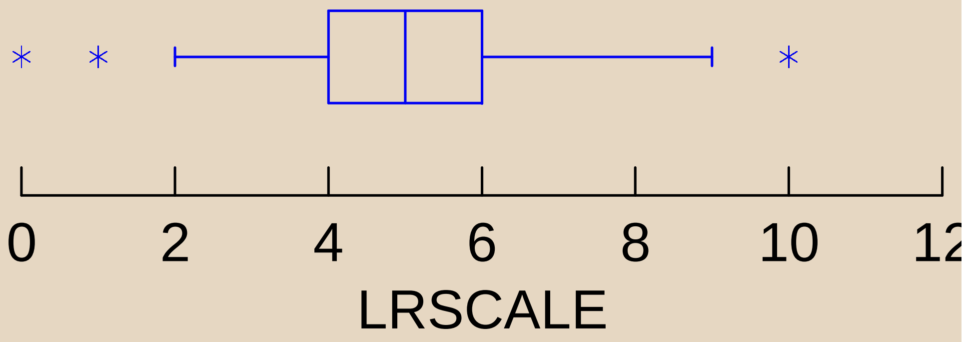
# Traditional approach

## First stage: synthesizing indicators at individual level

First level

Political placement on left-right scale

| Synthetic score (LRSCALE) | |
|---|---|
| Minimum | 0.0 |
| Maximum | 10.0 |
| Median | 5.0 |
| Mean | 5.1 |
| Standard Dev | 2.2 |
| Skewness | -0.0 |
| Kurtosis | 0.1 |




0 (left) – 10 (right)

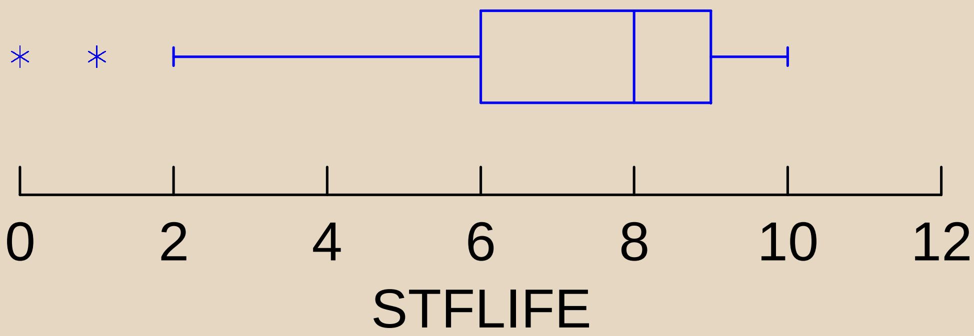
# Traditional approach

## First stage: synthesizing indicators at individual level

First level

how satisfied with life as a whole

| Synthetic score (STFLIFE) | |
|---|---|
| Minimum | 0.0 |
| Maximum | 10.0 |
| Median | 8.0 |
| Mean | 7.0 |
| Standard Dev | 2.3 |
| Skewness | -1.0 |
| Kurtosis | 0.7 |




0 (extremely dissatisfied) – 10 (extremely satisfied)

# Traditional approach

## First stage: synthesizing indicators at individual level

Second level

In order to synthesise the identified indicators, the traditional approach counts on the application of a synthesis technique (e.g., PCA).

In our case, the PCA results did not allow any meaningful synthesis since it produced two components on four indicators (!!!)

# Traditional approach

## Second stage: defining macro-units

**Goal**: synthesizing indicators observed at individual level in order to ascribe a synthetic value to groups.

The aggregation can be done through

- *additive* approach: a single value synthesizes the values observed at micro level (also through further indicators aggregation processes “second-level indicators aggregation”);
- *compositional* approach: when micro-units’ macro-units’ values are obtained by aggregating individual values in a certain number of homogeneous sub-groups

In our case, we adopted the latter approach in order to simultaneously aggregate indicators and cases.

# Traditional approach

Country level of acceptance

| Country | Acceptance mean score |
|---|---|
| AT | 2.61 (rank → 8) |
| BE | 2.41 (rank → 5) |
| CH | 2.18 (rank → 1) |
| CZ | 2.46 (rank → 6) |
| DE | 2.32 (rank → 3) |
| DK | 2.31 (rank → 2) |
| ES | 2.38 (rank → 4) |
| FI | 2.53 (rank → 7) |
| Overall | 2.42 |

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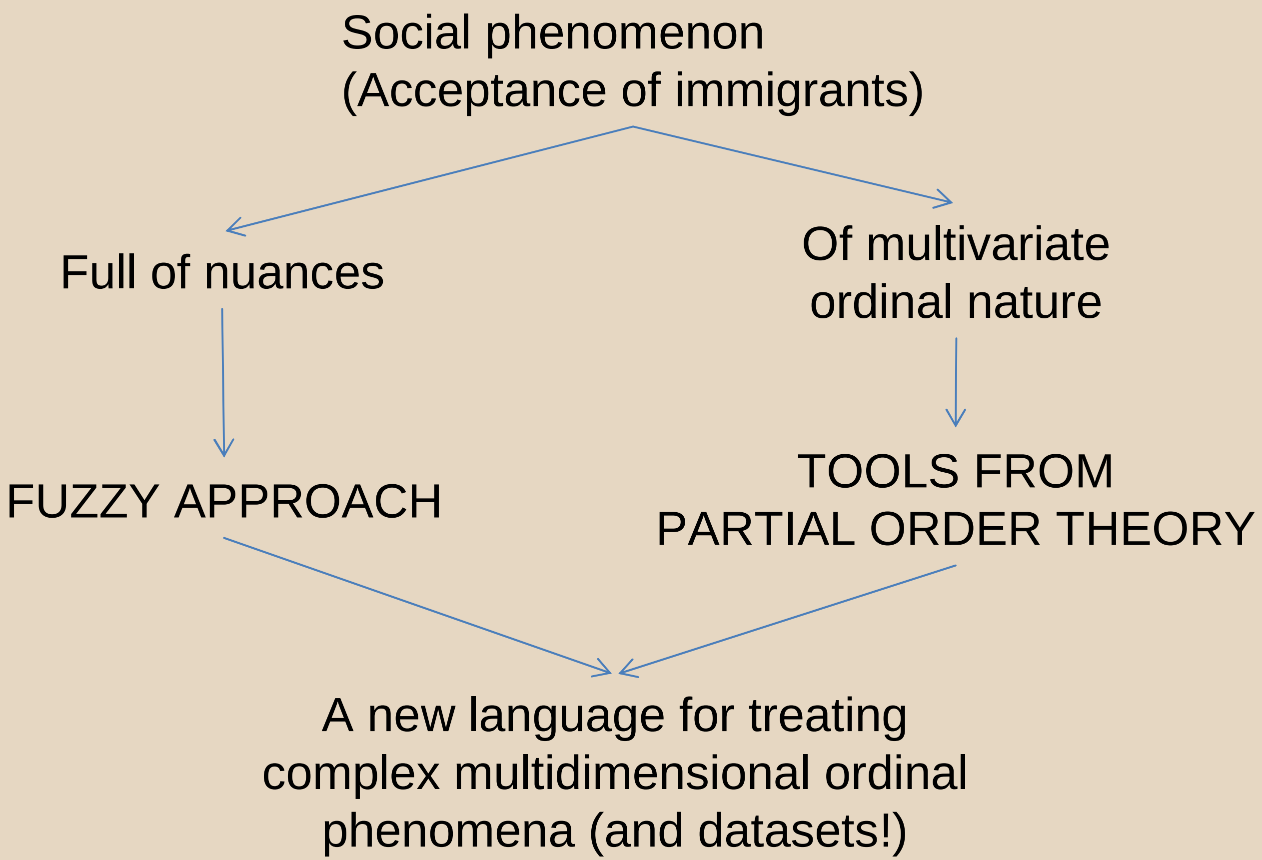
# Alternative approach

## Searching for new formal languages…



# Alternative approach

## Partial order analysis through a simple example

Many ordinal variables recorded on a population,

⇩

individuals cannot be directly ordered,

since each variable is likely to induce different rankings

The most natural way to represent such data is through a partial order.

# Alternative approach

## Partial order analysis through a simple example

We introduce basic concepts using a simple example, based on considering two variables from the European Social Survey, namely variables D4 (IMSMETN) and D5 (IMDFETN):

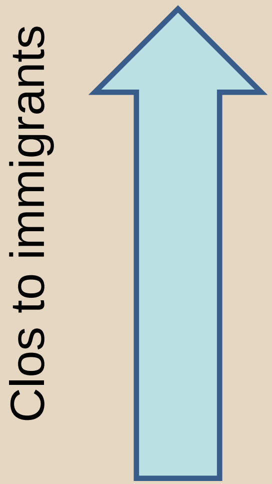
- D4: acceptance of many/few immigrants of same race/ethnic group as majority;
- D5: acceptance of many/few immigrants of different race/ethnic group from majority.

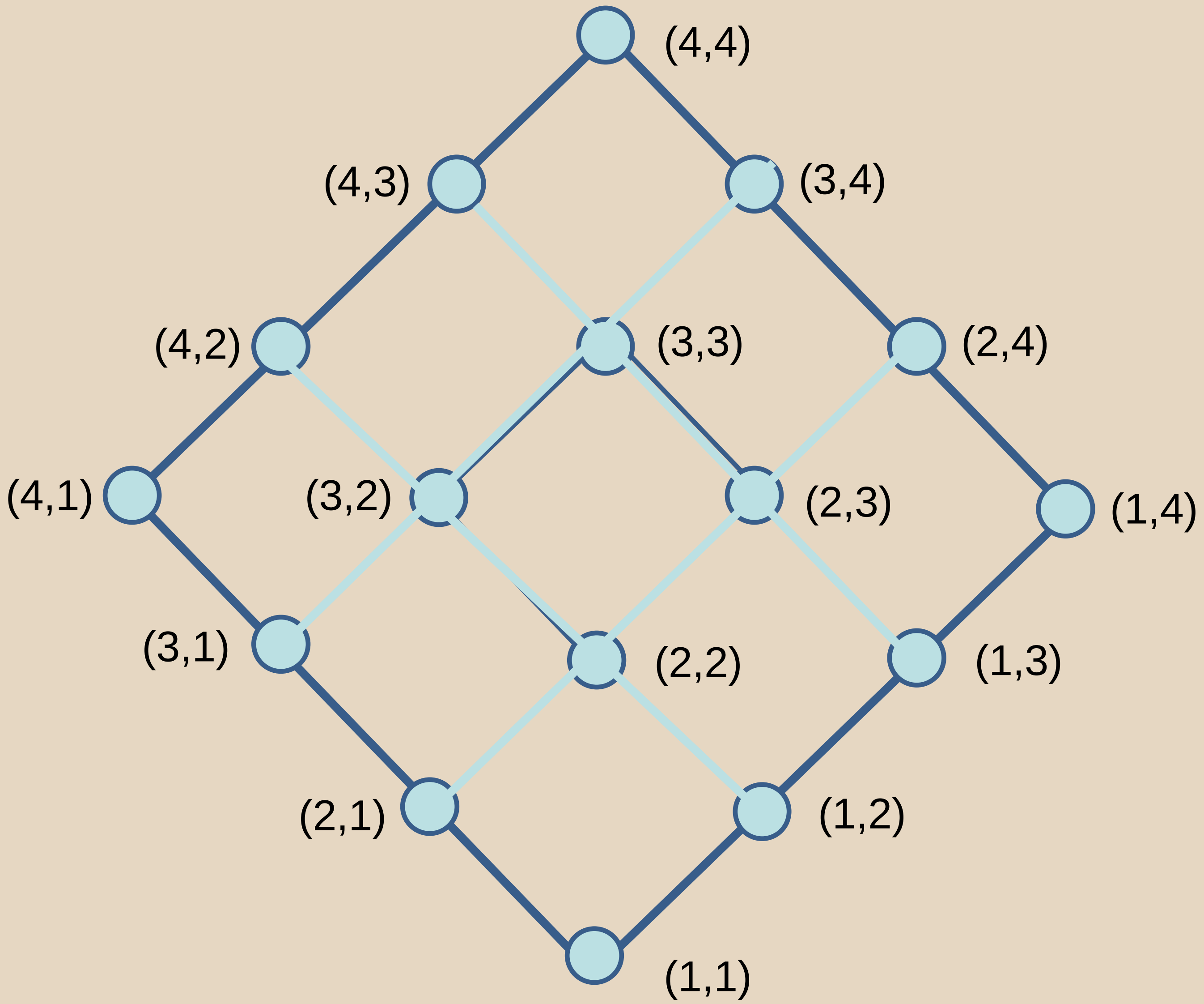
Both variables are recorded on a four grade scale from 1 (allow many) to 4 (allow none).

# Alternative approach

## Acceptance configurations on D4 and D5

example:
(4,2) stands for
(D4=4,D5=2)



# Alternative approach

## Assessing the degree of acceptance

It is clear that if $(a,b) \leq (c,d)$, then the degree of rejection of immigration of $(c,d)$ is greater than that of $(a,b)$. But:

- Is it possible to assess to what extent it is greater?
- Is it possible to assign to each configuration the corresponding degree?

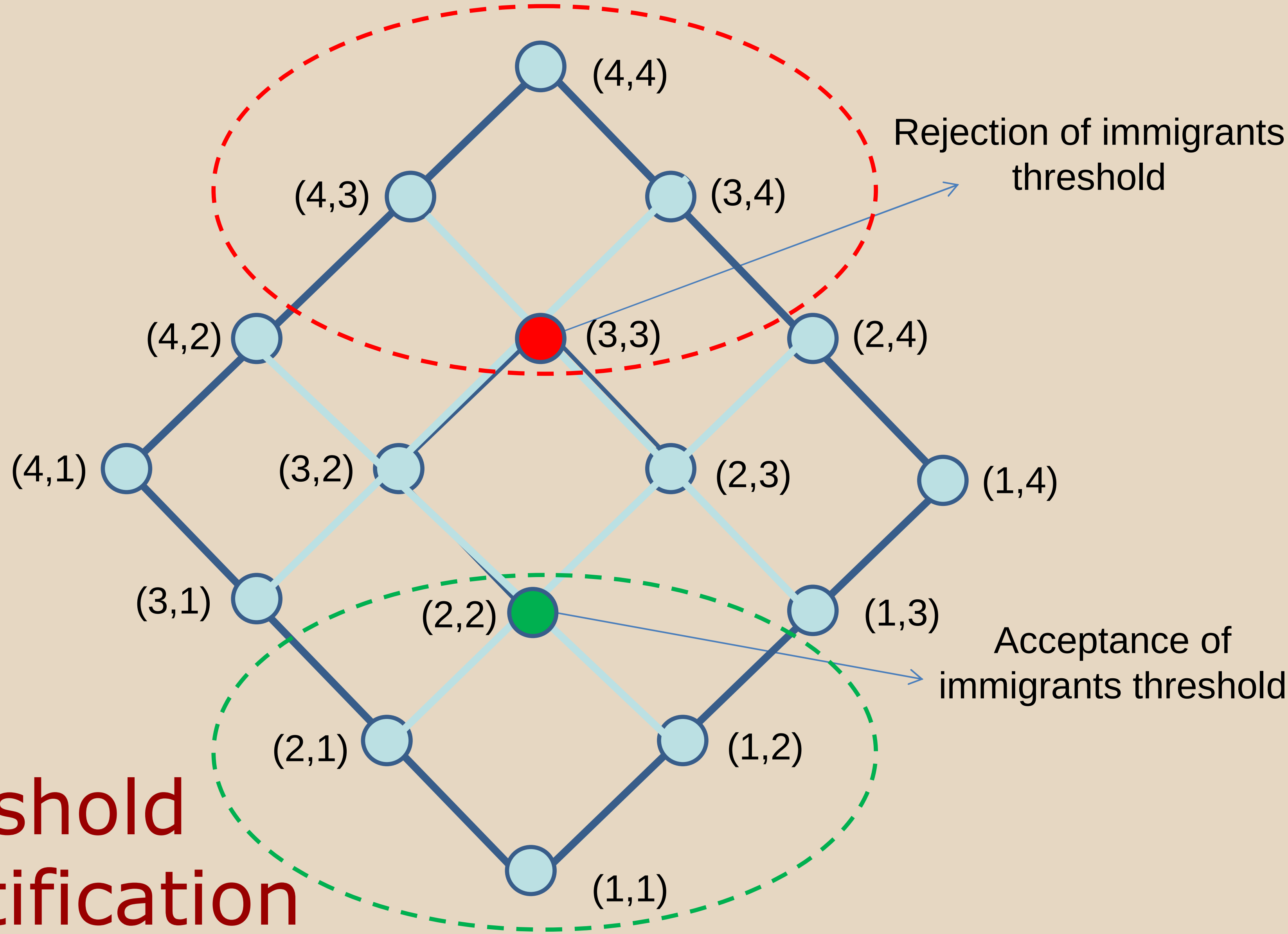
# Alternative approach

## Assessing the degree of acceptance

Yes, if we suppose that some configurations are identified as definitly representing closeness to immigrants or acceptance of immigrants, that is, if suitable rejection and acceptance thresholds are identified.

Here subjectivity enters but all the implications of the choice of such thresholds are then derived based only on the data structure.

# Alternative approach




Threshold identification

# Alternative approach

## Assessing the degree of acceptance

- Let us agree, in a fuzzy evaluation perspective (so as to take explicitly into account nuances), that nodes in the red ellipse has degree of rejection of immigration equal to 1 (the maximum) and that nodes in the green ellipse has degree of rejection of immigration equal to 0 (i.e. the minimum).

- All other nodes should receive a degree of rejection between 0 and 1, reproducing the ambiguities in the phenomenon.

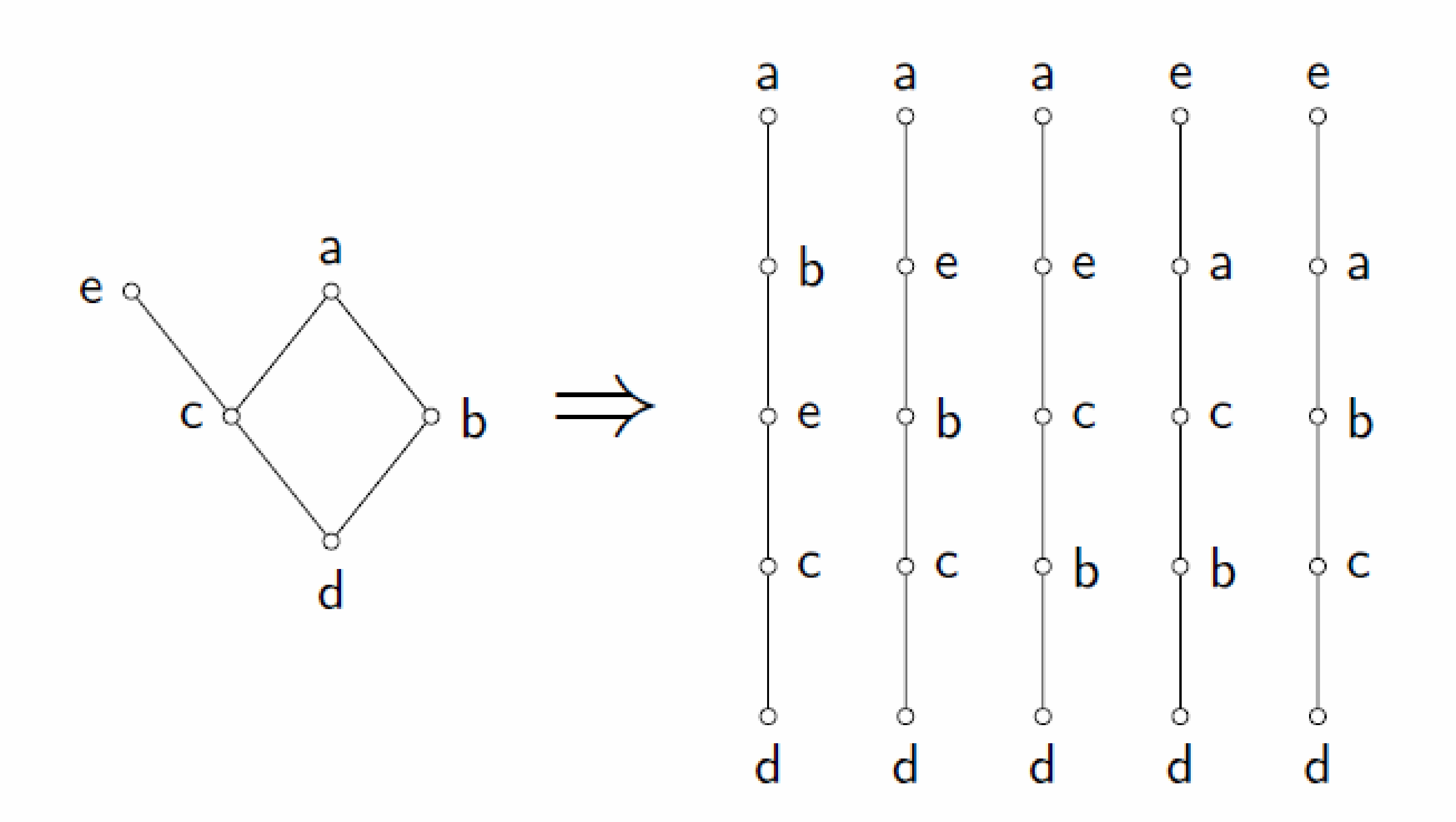
# Alternative approach

## Assessing the degree of acceptance

- The computation of such degrees is based only on the analysis of the partial order structure of the poset, that is, it is based on the analysis of the different relational position of each node, with respect to the thresholds selected.
- The required information about the degree of acceptance/rejection of immigration is extracted from the structure of the poset and not on the aggregation of variable scores (which are treated as they are, i.e. as ordinal variables).

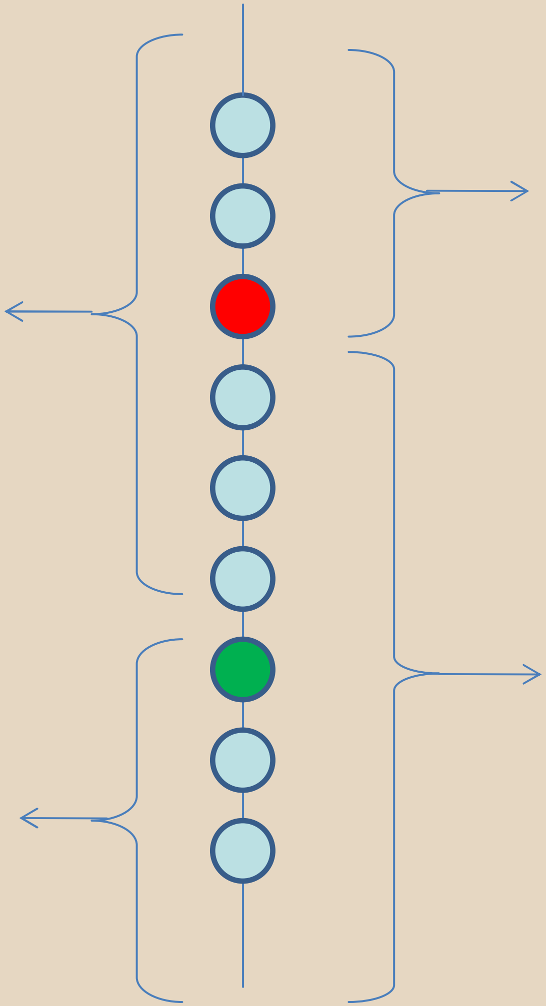
# Alternative approach

## Linear extensions of a poset



# Alternative approach

## The basic idea: pick up a linear extension …



These states receive degree of closeness to immigrants equal to 1.

These states receive degree of immigrant acceptance equal to 0.

These states receive degree of closeness to immigrants equal to 0.

These states receive degree of immigrant acceptance equal to 1.

# Alternative approach

## The basic idea

- In the end, for each state we get two degrees:
  - the first measures to what extent it can be classified as belonging to the group of states representing people who do not accept immigrants; → **deg1**
  - the second measures to what extent it can be classified as belonging to the group of states representing people who do accept immigrants. → **deg2**
- Turning **deg1** into **1-deg1** we get an alternative measure of acceptance of immigrants (in terms of non-rejection of them).

# Alternative approach

## The basic idea

- So, given the
  - *rejection threshold* (red) and
  - *acceptance threshold* (green)

  we get two different assesments of the degree of acceptance of immigrants, corresponding to each node.
- To get the final degree of immigration acceptance, for each node we compute the average of **1-deg1** and **deg2** (it can be shown that this is the only way to get a mathematically consistent fuzzy assessment of acceptance degree, out of the two "original" assessments).

# Alternative approach

## Results

| State | Acceptance degree<br>Acceptance threshold (2,2) | Rejection degree<br>Rejection threshold (3,3) | Final acceptance degree |
|---|---|---|---|
| (1,1) | 1,00 | 0,00 | 1,00 |
| (1,2) | 1,00 | 0,00 | 1,00 |
| (1,3) | 0,71 | 0,00 | 0,86 |
| (1,4) | 0,42 | 0,09 | 0,66 |
| (2,1) | 1,00 | 0,00 | 1,00 |
| (2,2) | 1,00 | 0,00 | 1,00 |
| (2,3) | 0,00 | 0,00 | 0,50 |
| (2,4) | 0,00 | 0,39 | 0,31 |
| (3,1) | 0,71 | 0,00 | 0,86 |
| (3,2) | 0,00 | 0,00 | 0,50 |
| (3,3) | 0,00 | 1,00 | 0,00 |
| (3,4) | 0,00 | 1,00 | 0,00 |
| (4,1) | 0,42 | 0,09 | 0,66 |
| (4,2) | 0,00 | 0,39 | 0,31 |
| (4,3) | 0,00 | 1,00 | 0,00 |
| (4,4) | 0,00 | 1,00 | 0,00 |

# Alternative approach

Country level of acceptance

| Country | Acceptance degree (D4 and D5) |
|---|---|
| AT | 0.49 (rank → 7.5) |
| BE | 0.62 (rank → 4) |
| CH | 0.74 (rank → 1) |
| CZ | 0.52 (rank → 6) |
| DE | 0.65 (rank → 2) |
| DK | 0.63 (rank → 3) |
| ES | 0.54 (rank → 5) |
| FI | 0.49 (rank → 7.5) |
| Overall | 0.62 |

# Alternative approach

## A more complex example

- A similar analysis has been performed on the four variables D6, D7, D8, D9.
- The resulting poset has 4x4x4x4=256 states and cannot be depicted.
- The red (*rejection*) threshold has been identified as the state (2,3,2,3).
- The green (*acceptance*) threshold has been identified as the state (2,2,2,2) .
- In both cases, the thresholds have been identified for explanation purposes. A more meaningful choice requires expert's judgment and/or further analysis.

# Alternative approach

Country level of acceptance

| Country | Acceptance degree (D6, D7, D8, D9) |
|---|---|
| AT | 0.33 (rank → 8) |
| BE | 0.52 (rank → 3) |
| CH | 0.64 (rank → 1) |
| CZ | 0.50 (rank → 4) |
| DE | 0.53 (rank → 2) |
| DK | 0.49 (rank → 5) |
| ES | 0.48 (rank → 6) |
| FI | 0.37 (rank → 7) |
| Overall | 0.48 |

(given the thresholds)

# Alternative approach

## Comments

- We have given a brief example of how poset theory can be used to compute social indicators out of ordinal data, without turning them into numerical scores.
- Due to the exemplificative nature of the slides, the computed numbers should be taken just as rough measures. They depend upon the choice of the thresholds and some sensitivity analysis should be added.
- The poset describing variables D4 and D5 is very small (for presentation purposes), so the variability of the acceptance degrees over its 16 nodes is small compared to that of nodes in the poset concerning D6-D7-D8-D9 (that comprises 256 nodes). Also for this reason, the final numbers obtained in the two cases are not directly comparable.

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# State-of-the-art and future perspectives

## State of the art

- This approach has been (and is being currently) applied to the study of material deprivation, based on EU-SILC data. We are planning to study also other social phenomena.
- Approximated analytical formulas are being developed, so that the computations involved in this approach can be performed without relying on heavy and complex numerical algorithms.
- It is already possible to define thresholds composed of more than a single node. This makes the proposed approach more flexible to real situations.

# State-of-the-art and future perspectives

## Future perspectives

- Integration of poset analysis and Structural Equation Modeling.
- Definition of algorithms to help identifying thresholds.
- Definition of "weighting" schemes for ordinal variables, i.e. of a way to take into account the different relevance of different variables, without introducing numerical weights.
- Definition of clustering algorithms, for reducing the dimension of posets, when the number of variables and/ot the number of possible scores for each variable is high.

In practice: developing a full set of analytical tools for dealing with ordinal variables in a consistent way.

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