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# Measuring multidimensional polarization with ordinal data

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# The issue(s)

## **Social perspective**

- The «disappearing of the middle-class» and the transformation of the structure of our societies, with possible effects on the properly functioning of our democracies;
- Polarization is not confined to income, but involves several aspects of societal life;
- To realize the existence of complex socio-economic dynamics and to overcome «flat» policies in favour of different actions for different needs.

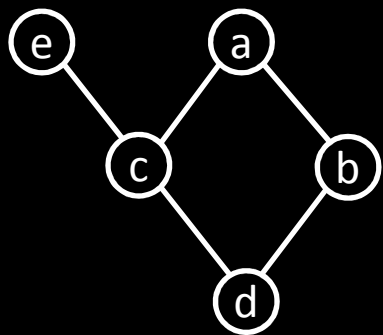
## **Statistical perspective**

- Dealing with ordinal variables: a problematic issue in synthetic indexes computation;
- No literature (but a few papers) on measuring ordinal multidimensional polarization;
- Relevance of producing «faithful» indicators to answer decision-makers' needs.

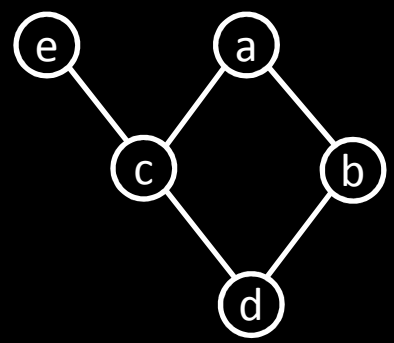
# Goal of the presentation

- To suggest a new way to address the problem of measuring polarization in a multidimensional ordinal setting;
- To provide a first simple application on real data.

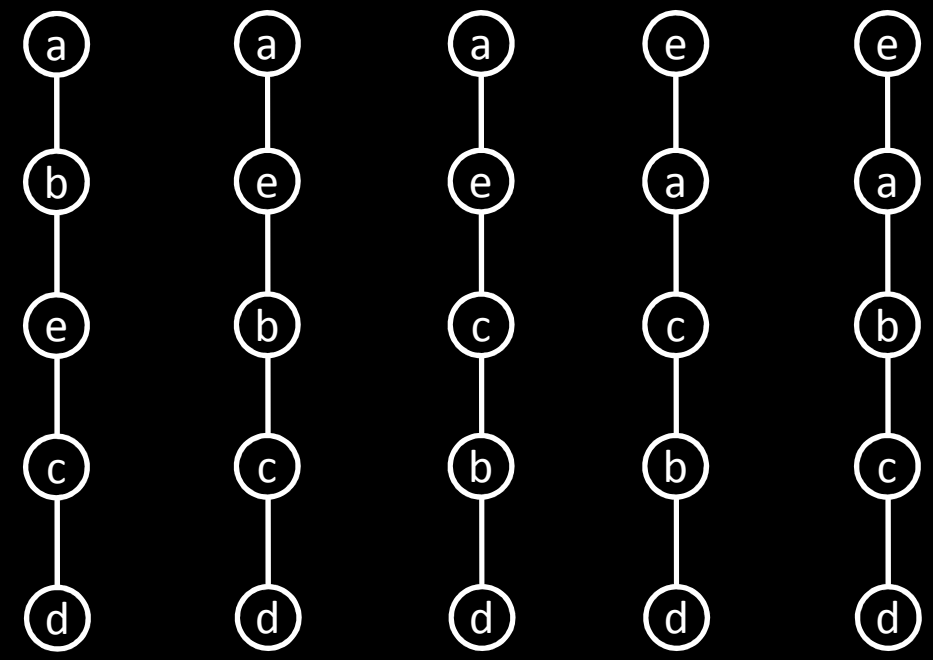
# A poset



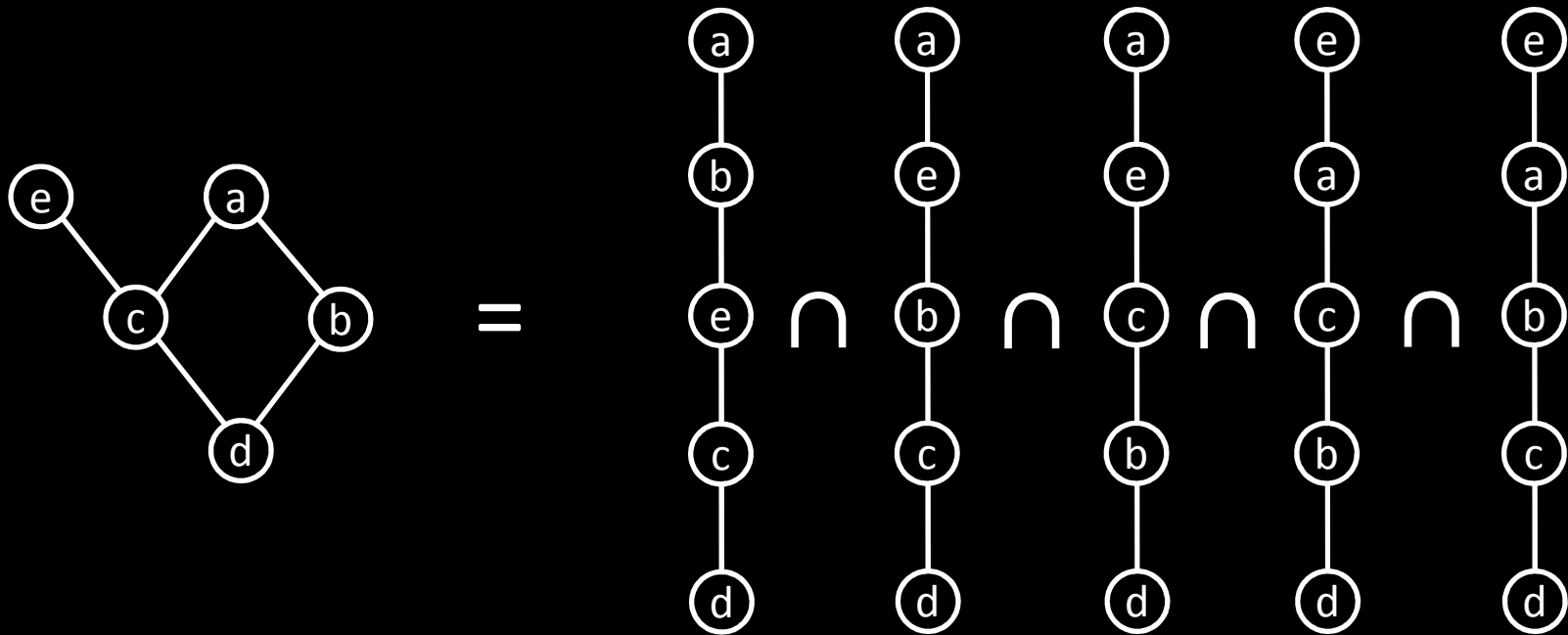
# Linear extensions



$\cup$



# A fundamental relationship



# Driving example: data

Three binary variables (1 - good; 0 – bad) pertaining to well-being in Italy, from Indagine Multiscopo\* 2010:

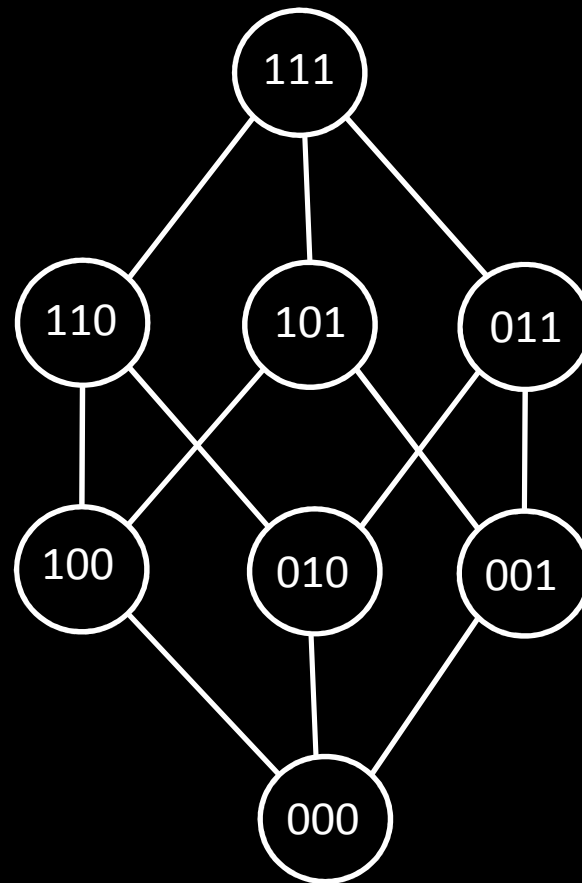
1. Subjective health status;
2. Satisfaction for one's own economic status;
3. Satisfaction for quality of leisure time.

Profile	Count
111	14207
110	4255
101	1291
011	9282
100	807
010	5876
001	2400
000	2847

\*Data are available within a research protocol between Istat and University of Florence: "Quality of Life in Italy: assessment through data from Multipurpose Survey about Families Aspects of daily life".

# Poset on 3 binary variables

## Hasse diagram





# Approaches to multidimensional polarization measurement

## Classical

Unidimensional polarization axioms directly extended to the multidimensional case (often with cumbersome results)

-> Multidimensional polarization measure computed as functions (usually means) of unidimensional polarization on single variables.

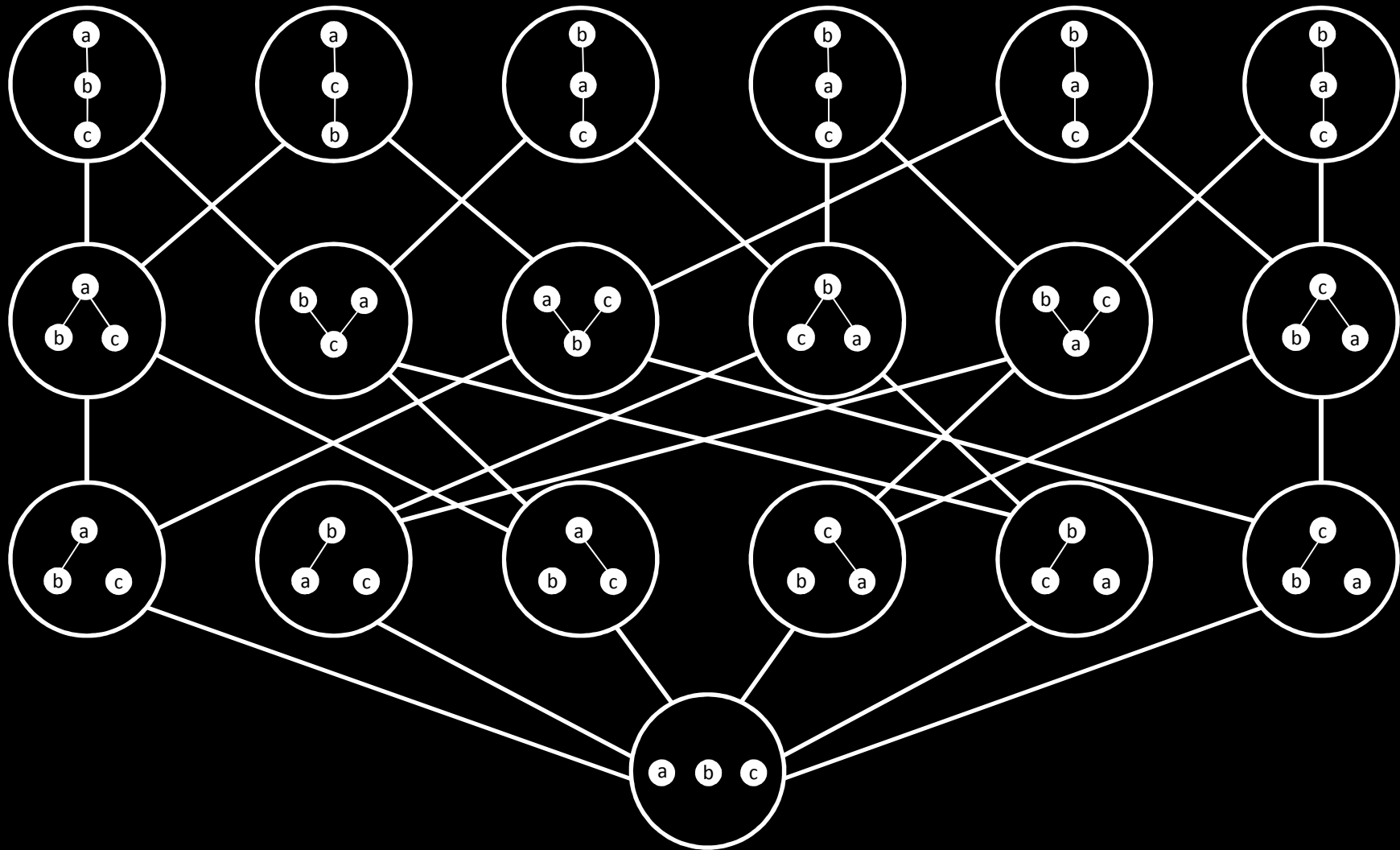
## Poset

Search directly for axioms to build multidimensional polarization indexes out of unidimensional ones.

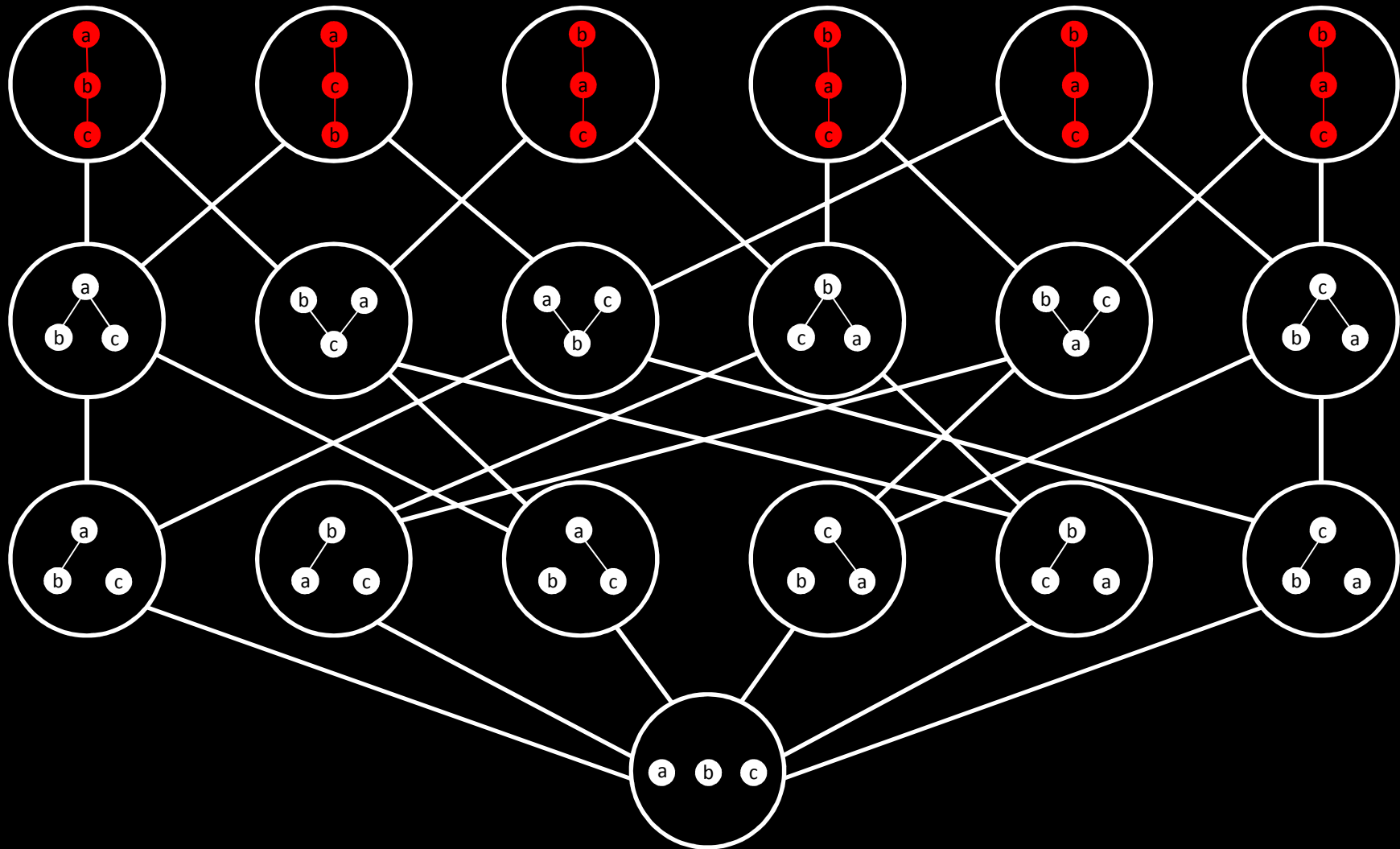
Based on combinatoric – algebraic concepts (discrete mathematics)

-> Multidimensional polarization measure computed as functions (usually means) of unidimensional polarization on single linear extensions.

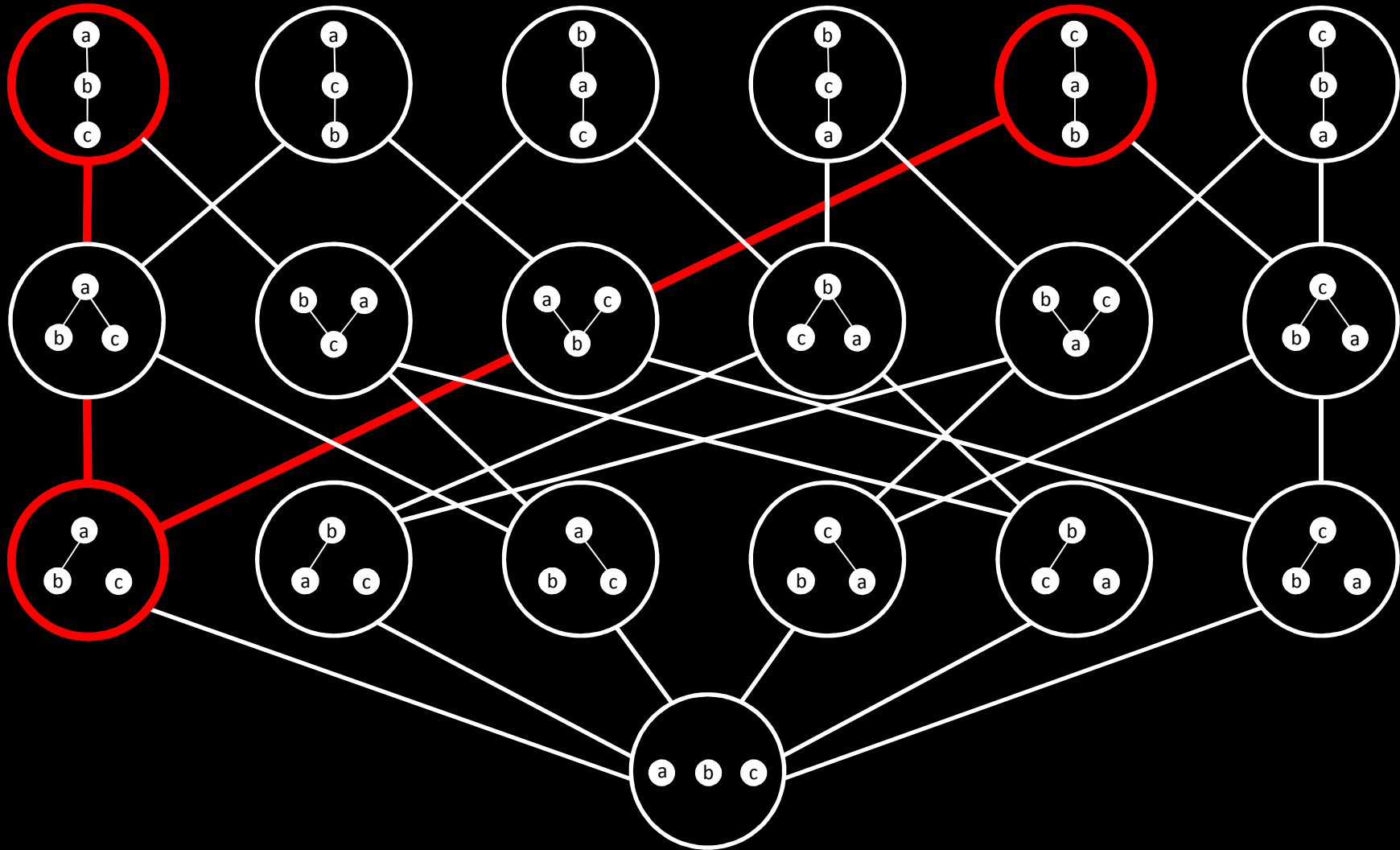
To give an idea  
(semilattice)



# Generating elements



# Intersection (meet)



## The basic idea

Given a polarization a unidimensional polarization index defined on linear extensions, extend it to the whole semilattice.

# Mathematical path to a new axiomatic approach

Associativity of the semilattice

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Theory of means by Kolmogorov-Nagumo-de Finetti



Multidimensional polarization measures as means of measures of linear  
extensions

## Multidimensional polarization index with ordinal variables

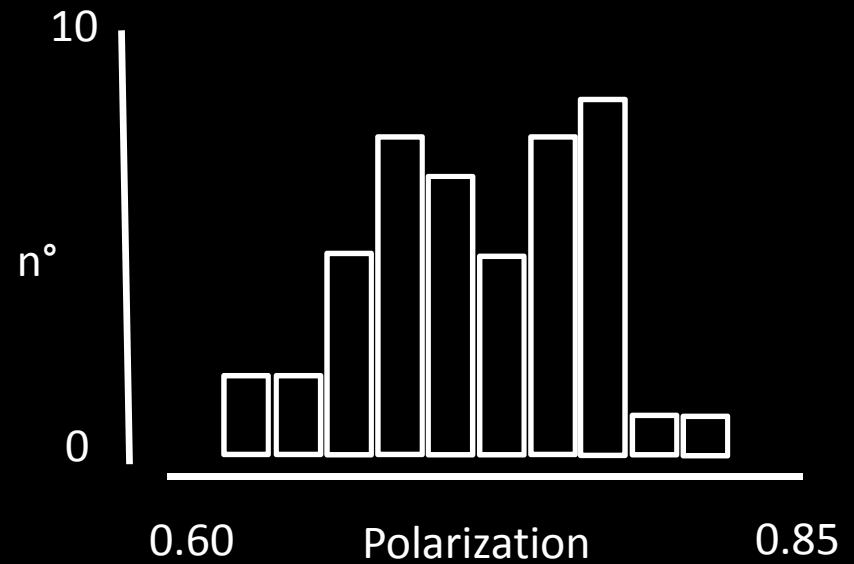
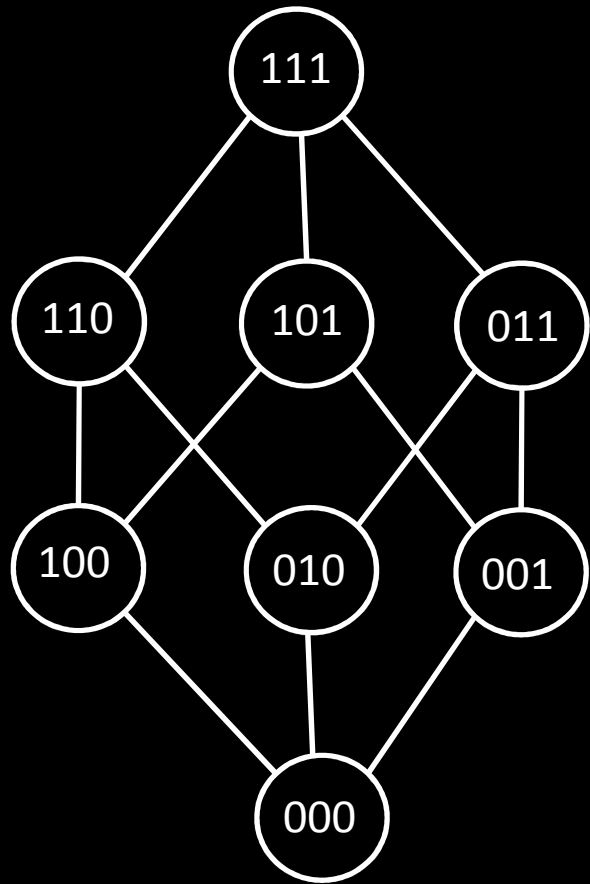
There are many indexes to measure polarization (or inequality) on ordinal variables.

In this attempt, we have chosen the normalized form of the following

$$I(\ell) = \sum_{i < j}^8 n_i n_j (j - i)$$

where  $\ell$  is a linear extension,  $j$  and  $i$  are the ranks and  $n_i$ ,  $n_j$  the number of elements in the profiles ranked  $i$  and  $j$  respectively.

# Distribution of polarization on its 48 linear extensions and average



Multidimensional  
Polarization (average) **0.72**



# References

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